Provincial Kindergarten to Grade 4 Numeracy Screening Assessments

Interpretation Guide

Heather Douglas, Ph.D., B.Ed. and Jo-Anne LeFevre, Ph.D.
Department of Cognitive Science
Carleton University

Provincial Kindergarten to Grade 4 Numeracy Screening Assessments Interpretation Guide

Provincial Kindergarten to Grade 4 Numeracy Screening Assessments Interpretation Guide

The Provincial Numeracy Screening Assessments are based on EarlyMathAssessment@School (EMA@School), which is a set of tasks that measures children and students' developing mathematical knowledge in Kindergarten to Grade 4. Specifically, the screeners capture student's knowledge of symbolic numbers. The screeners were designed at the Carleton University Centre for Applied Cognitive Research and are rooted in current theory on mathematical cognition and development. Most tasks for students in Grades 1-4 can be administered in group settings, including whole classes, using paper and pencil. The Kindergarten and Grade 1 counting tasks are administered one-on-one. Importantly, the results can be used to help teachers identify and address gaps in their students' foundational understanding of numbers.

The Theory Behind EarlyMath@School

Multiple cognitive processes are involved in learning math. The *Pathways to Mathematics Model* describes the cognitive skills that contribute to mathematical learning and thinking: attention, relational reasoning, language, and quantity skills (Di Lonardo Burr et al., in 2022; LeFevre et al., 2010; Sowinski et al., 2015). Attentional skills include working memory and executive function. Working memory involves holding and manipulating information: for example, keeping track of numbers in a count sequence or adding numbers using mental math (DeStefano & LeFevre, 2004). Executive functions involve inhibitory control, cognitive flexibility, and updating (Friedman & Miyake, 2017). For early mathematics, these skills are involved in counting objects, comparing sets, and selecting math-relevant information from the environment. Attentional skills are correlated with mathematics for children and adults (Cowan et al., 2011; LeFevre et al., 2005; Peng et al., 2016; Raghubar et al., 2010).

Relational reasoning involves skills such as identifying, extending, or filling in a missing item in a pattern. Notably, children's early patterning skills predict their later number knowledge (Di Lonardo Burr et al., in 2022; Zippert et al., 2020). Patterning may support math learning because mathematics involves understanding the interrelations between numbers and the rules that govern those interrelations. Relational reasoning skills continue to support mathematical learning beyond Kindergarten (Kalra et al., 2020).

Language skills also play an important role in math learning (Peng et al., 2020; Zhang & Lin, 2015). Teachers and parents communicate mathematical ideas with words. From preschool onwards, children's general vocabulary skills predict their later mathematical achievement (LeFevre et al., 2010; Purpura & Ganley, 2014; Xu et al., 2021). Vocabulary that is specific to math such as *sum*, *equals*, or *greater than* is also an important aspect of math learning (Hornburg et al., 2018; Purpura & Reid, 2016; Toll & Van Luit, 2014). For example, preschoolers' knowledge of spatial words (e.g. *near*, *far*, *left*, *right*) and quantity words (e.g., *more*, *less*, *larger*, *smaller*) predicts their success on tasks such as verbal counting, number identification, cardinality, number ordering, and word problems (Hornburg et al., 2018). Children start Kindergarten with some knowledge of math words; this math vocabulary is relevant for connecting children's informal math knowledge with formal number knowledge (Purpura et al., 2013). A descriptive model of how cognitive and mathematical processes are related is shown in Figure 1. Number knowledge is built using these cognitive foundations.

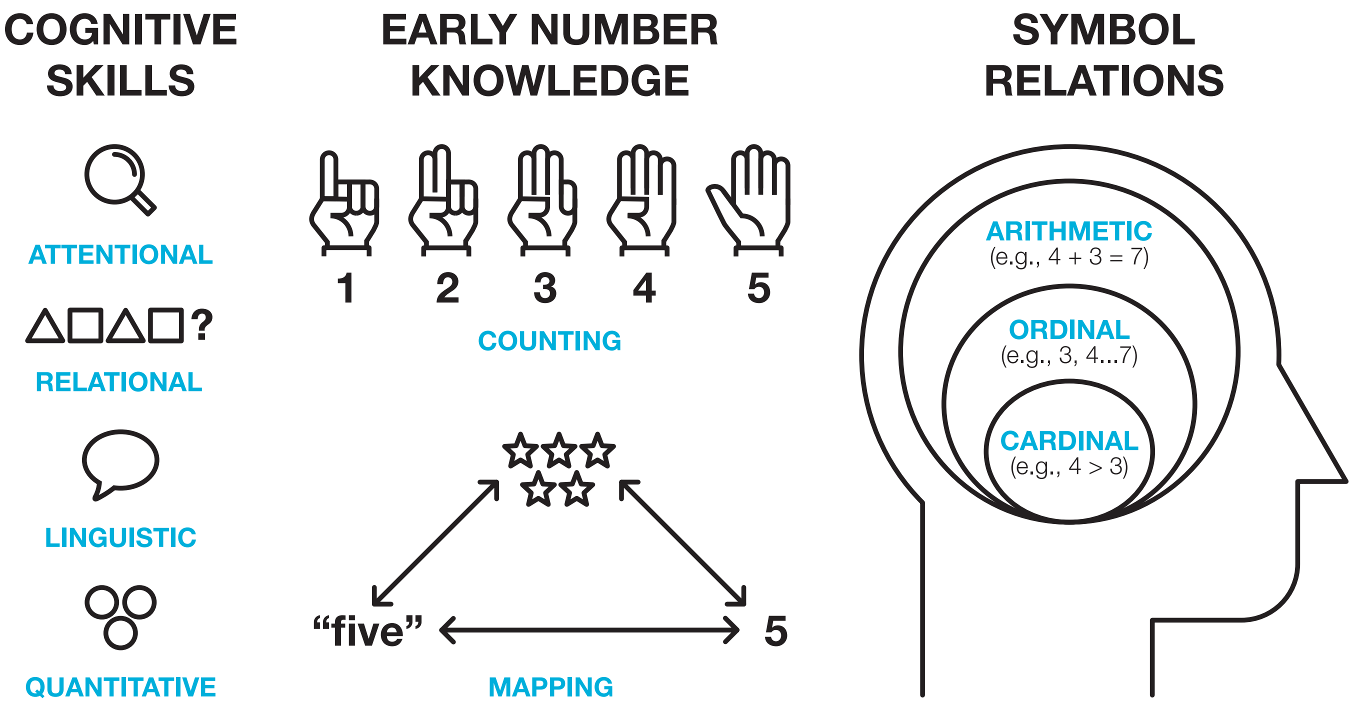

Figure 1. Early Mathematical Learning

In summary, attentional, relational reasoning, and linguistic skills support children’s developing number knowledge. Very basic quantitative skills, such as the ability to quickly identify small numbers of objects without counting (i.e., subitizing), are also fundamental to early mathematical learning (Carey & Barner, 2019). When they enter school, children will use cognitive skills and the informal mathematical knowledge they gained at home or in preschool to develop the specific knowledge about numbers that supports further learning (Purpura et al., 2015; Purpura & Longigan, 2015; Siegler et al., 2012). Thus, the screeners focus directly on children’s developing number knowledge. We assume that the most direct route to improving children’s skills in Kindergarten to Grade 4 is to identify and focus on core number knowledge: counting, number identification, number patterns, number lines, arithmetic, and, for the Kindergarten level, links between quantities and number symbols.

Components of the Provincial Numeracy Screening Assessments

Foundational early mathematical knowledge includes number knowledge, number relations, and number operations (Devlin et al., 2022). These screeners include tasks to measure each of these subdomains. Skills in each subdomain are related, but each predicts mathematics separately. The packages are designed to measure the number skills that form the building blocks for mathematical development. One challenge was to select tasks that would be suitable across grades and allow growth across the year. Continuity was used by modifying time limits, stimulus selection, and in some cases, specifying different tasks at different grade levels when knowledge profiles varied with grade (Chard et al., 2005). For children in Kindergarten, these tasks include counting skills, early number knowledge, and basic number symbol relations. For older students, the tasks focus on their knowledge of large numbers, their developing understanding of the interrelations amongst number symbols, and number operations. The tasks adapted from the EMA@School are all well known in the research literature and many appear in some form on other screeners. We also were constrained by the knowledge that the screeners would be administered by teachers, so the implementation had to be sensitive to the demands of classrooms.

Number Knowledge

Counting is one of the earliest number skills. However, some students arrive at school with extensive counting skills whereas others are unable to count successfully to 5 (Litkowski et al., 2020). Learning to count involves more than just reciting the count sequence, however. Children need to learn the purpose of counting, which is to determine quantity (Gray & Reeve, 2014, 2016). To count with meaning, children need to understand three essential counting principles: a) stable order of the count words, that is, knowing and using the number words in order; b) one-to-one correspondence between the items counted and the count words, that is, counting each item once and only once; c) cardinality—when counting, the last number word said, tells how many items in a set (Gallistel & Gelman, 1990); and a non-essential principle—order irrelevance—when counting a group of items, the order in which they are counted doesn't matter. This is not acquired by children until mid-elementary school (Kamawar et al., 2010; LeFevre et al., 2006). Learning the counting principles occurs gradually (Purpura et al., 2013). Thus, to identify children's developing counting knowledge, the *Counting* tasks measure children's knowledge of the count sequence and of the key counting principles in Kindergarten. The *Next Number* task requires that students complete a counting sequence. These sequences get progressively more complex, tapping into children's knowledge of the rules for generating higher numbers (e.g., 12 13 14 15 __ __; which numbers come next?), knowledge of other sequences (e.g., 2 4 6 8 __ __), and fluency of access, allowing them to count backwards (e.g., 10 9 8 7 __ __).

Learning is about creating connections. In math, creating connections between symbols is central to developing math knowledge. Math learning is cumulative, with new skills being built on existing skills. Thus, mathematical development can be viewed, at least in part, as a hierarchy of connections among numerical symbols (Hiebert, 1988; Núñez, 2017). At the base of the hierarchy is connecting the symbol with its concrete representation: for example, knowing that the number 4 represents a quantity of ♥ ♥ ♥ ♥. Before successfully connecting the digit 4 with the quantity ♥ ♥ ♥ ♥, however, children need to connect the verbal number word "four" with its corresponding number symbol (Hurst et al., 2017; Jiménez Lira et al., 2017). *Mapping* is the term used to describe connecting quantities, digits, and verbal number words. The *Naming Numbers* task, where Kindergarten children are asked to connect verbal number words with number symbols, is a measure of students' early mapping skills. The *Counting*, *Next Numbers*, and *Naming Numbers* screeners can be used to identify which students need extra support because they have not yet mastered these minimal standards.

As students learn about larger numbers (i.e., 10 to 100 to 1000) they need to understand new rules for mapping number words to number symbols. These rules reflect the structure of the number system, but there can be conflict between the verbal labels and the written symbols, especially in some languages. Consider quatre-vingt-dix-huit versus ninety-eight, for example. Even a familiar number like eighteen is often written as 81 when students are learning. In languages such as Dutch or German, where decade numbers such as 42 are named as "four-and-twenty", inversion errors are common into Grade 2 (Imbo et al., 2014). Beyond the verbal labels and the written symbols, children need to understand that the position of a digit indicates its value. Thus, 21 is two tens and one unit, whereas 12 is one ten and two units. Each year, the range of numbers that students learn increases. By Grade 2, for example, students are expected to know the number system in the hundreds. A student who hears "two-hundred five" and writes "2005" has not yet mastered the rules for transcoding spoken numbers into written digits (Skwarchuk & Anglin, 2002). This error (called a syntactic error) indicates the student does not understand how the relative positions of written numbers reflect place value. Transcoding between verbal and written numbers is related to arithmetic skills (Clayton et al., 2020; Simmons et al., 2012; Sowinski et al., 2015). Children with low math performance demonstrate poor understanding of place value rules at the number size they should have learned (Moura et al., 2013). The *Number Writing* transcoding task helps teachers identify students' transcoding errors (Grades 1 to 4). Specifically, teachers can identify gaps in knowledge of larger numbers (e.g., literal translations such as writing two hundred five as 2005 or reversing number order; 28 versus 82, etc.). In response, teachers can customize number writing supports as needed, using number matching games, place-value charts, and other activities that highlight the patterns and rules that determine number structure.

Number Relations

Numbers can be related to each other in many different ways. Consider the numbers 1, 2, and 3. These numbers have cardinal (quantity) relations (e.g., 3 > 1; 2 < 3), ordinal relations (e.g., 2 comes after 1 and before 3), and arithmetic relations (e.g., 1 + 2 = 3; 3 – 1 = 2). As children acquire various associations among symbolic numbers, including cardinal, ordinal, and arithmetical connections, these associations form an increasingly interconnected mental number network (Hiebert, 1988; Siegler & Lortie-Forgues, 2014; Xu et al., 2019; Xu & LeFevre, 2020). As shown in Figure 1, more advanced arithmetic associations are built on basic cardinal and ordinal associations. Fast and accurate access to these number associations indicates that students are making progress in developing their mental network. Teachers can measure how efficiently students access the cardinal, ordinal, and arithmetic relations amongst numbers using the *Comparing Numbers*, *Ordering of Numbers*, and *Numbers on the Number Line* tasks.

Ansari and his colleagues have shown that number comparison is a foundational skill (Hawes et al., 2019; Nosworthy et al., 2013), and have claimed that fast and accurate number comparison is "… as important to math as phonological awareness is to reading" (Vanbinst et al., 2016). *Comparing Numbers* combines knowledge of how symbols are connected to quantities with the ability to compare those quantities mentally. Number comparison can be used to identify children with persistent developmental dyscalculia (Bugden et al., 2020).

The *Numbers on the Number Line* task requires students to use their number skills to judge proportional relations among numbers. Thus, the *Numbers on the Number Line* task (see Figure 2), in a number range appropriate to the students' number knowledge (e.g., 0-1, 0-10, 0-100, 0-1000, 0-10 000), taps into relative magnitude, spatial skills, and a preliminary understanding of rational numbers (Bailey et al., 2014; LeFevre et al., 2013). Number line performance is a sensitive index of developing mathematical knowledge (Booth & Siegler, 2006).

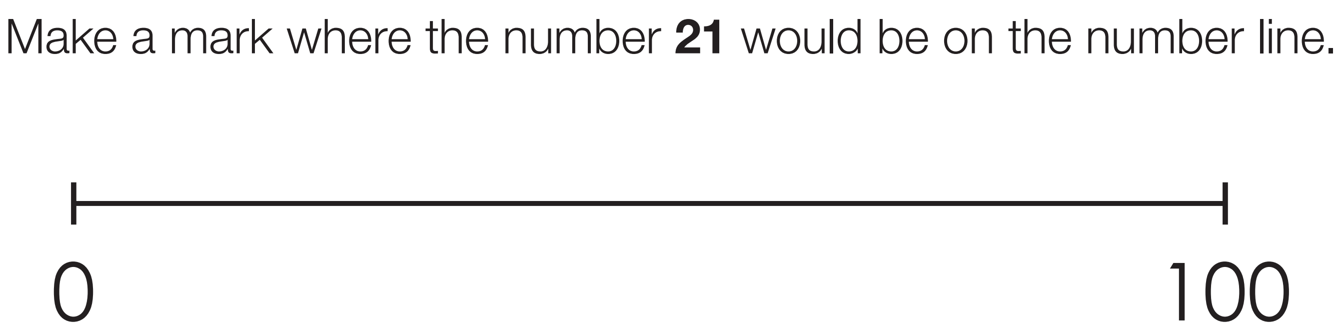

Figure 2. The *Numbers on the Number Line* Task

Students enter school with a set of basic cognitive skills and some early number knowledge. Student's mathematical skills are built on these early competencies. The Numeracy Screener package is designed to measure the number skills that form the building blocks for mathematical development. For children in Kindergarten, this includes tasks of their early number knowledge and basic number symbol relations. For older students, the tasks focus on their developing understanding of the interrelations amongst number symbols.

Tasks and Interpretations

Most Numeracy Screening tasks for students in Grades 1-4 can be administered in group settings, including whole classes, using paper and pencil. The Kindergarten tasks and the Grade 1 counting and naming numbers tasks are administered individually. These tasks measure number knowledge, number relations, and number applications, as described below.

Number Knowledge

For Kindergarten and Grade 1 students, teachers measure core counting skills within three measures: knowledge of the count sequence (**Rote Counting** and **Next Numbers**), and one-to-one correspondence plus cardinality (**Counting Sets**). Students also complete a **Naming Numbers** task where they are asked to match verbal number words with digits. By the end of Kindergarten, children are expected to master the counting principles and know their numbers to 10. Thus, the tasks can be used to identify which children need extra support. Such support might include pointing and counting practice (for one-to-one correspondence), digit matching games (for mapping and cardinality practice), counting songs, and other applicable activities. Because these skills are foundational for later learning, teachers can also use these tasks (Rote Counting, Counting Sets, Next Numbers, and Naming Numbers) to identify and intervene with students who are struggling to master more advanced concepts.

Older students (Grades 1 to 4) complete the number transcoding task in which they write multi-digit numbers in response to spoken number words. Students in Grades 1 are expected to know numbers to 100, whereas students in Grade 2 are expected to know numbers to 1 000. Students in Grade 3 and entering Grade 4 are expected to know numbers to 100 000. Teachers can use this task to identify the kinds of mistakes their students make. Specifically, teachers can identify where students struggle (i.e., writing 2-digit, 3-digit, or 4-digit numbers) and how they struggle (e.g., adding extra zeroes, reversing number order; 28 versus 82, etc.). In response, teachers can customize number writing supports as needed, such as using number matching games, place-value charts, and other applicable supports.

Number Relations

Measures of number relations can be used to identify the strength of students' number knowledge. Students first acquire fluent cardinal relations that reflect quantity. The **Comparing Numbers** task is a timed number comparison task where students quickly and accurately cross off the numerically larger digit in a pair. Quick and accurate access to quantities from symbolic numbers supports the development of other number skills like ordinal knowledge, arithmetic, and fractions. **Comparing Numbers** is used at all grade levels. Kindergarten children and Grade 1 students with the weakest number comparison skills lack strong links between the digit symbols and quantities. For these students, teachers can focus on activities that support linking digits and quantities such as card games like War, board games like Sorry, or dice games — activities where students practice linking visual symbols with quantities (Gasteiger & Moeller, 2021).

Knowledge of the sequential relations amongst numbers is distinct from knowledge about quantity (Lyons & Ansari, 2015; Lyons & Beilock, 2013). **Ordering of Numbers** is a timed task where students quickly and accurately judge if three digits are in increasing order (e.g., 2 3 5). In grades 2 and 3, students' fast and accurate access to number sequences captures the developing mental framework for number relations that supports calculation skills (Lyons et al., 2014; Sasanguie et al., 2017; Sasanguie & Vos, 2018). For students with weak order judgment skills, teachers can help students develop and use number ordering relations in various contexts such as games involving sequences – ordering numbers, what comes before, what comes after, and so on.

Estimating the position of a **Number on a Number Line** captures two aspects of a student's mathematical knowledge. First, it measures their understanding of the ordinal relations among numbers (e.g., 3 comes after 2 and before 4; 50 comes after 40 and before 60). Second, number line estimation requires proportional reasoning skills. Proportional reasoning is used to place the number accurately on the line (e.g., 49 is close to 50 and 50 is one-half of 100 so an appropriate strategy is to divide the line in half). Students with estimation errors greater than 20% (e.g., placing 7 either below 5 or above 9 on a 0–10 number line; placing all numbers above 100 near to 1000) have poor understanding of the number range. Teachers can support these students with activities that promote understanding of ordinal relations (e.g., what comes before, what comes next) (Xu & LeFevre, 2016), proportional reasoning (e.g., mark the middle of the number line, identify the middle number), linear board games (Siegler &

Ramani, 2009) and other applicable activities in the given number range.

Number Operations

Number Facts shows how quickly and efficiently students can retrieve number facts or apply efficient strategies. Built on knowledge of cardinal and ordinal associations, fluency skills scaffold higher-level mathematical competence. Importantly, arithmetic fluency is a core ability—it is both a predictor and a correlate of most other mathematical skills (Price et al., 2013). Accordingly, fluent access to math facts is important for problem solving (Lin, 2020). Students who know their number facts have more working memory available to strategize and focus on the problem they are solving as opposed to the arithmetic they need to solve the problem. Using the Numeracy Screening tasks teachers can identify students with weak arithmetic fluency, and they can gauge the current arithmetic skills of their group of students. Teachers can support fact fluency with adaptive online games, with card or board games (e.g., cribbage), classroom practice, and other applicable activities (Gasteiger & Moeller, 2021).

Conceptual knowledge of arithmetic principles is reciprocally related to procedural fluency (Rittle-Johnson & Alibali, 1999) and both types of knowledge are central to the development of strong mathematical understanding (Crooks & Alibali, 2014; Fyfe et al., 2012). Thus, children's acquisition of the basic number operations is a foundational predictor of later fraction and algebra skills (Schneider et al., 2017).

The **Equations** task measures students' knowledge of five principles: identity, negation, inversion, commutativity, and equivalence. Fluent access to arithmetic facts and procedures (for example, knowing that 3 + 1 is 4), is complemented by knowledge of the concepts or rules that define number relations (Knuth et al., 2006). Procedural and conceptual knowledge develop reciprocally (Rittle-Johnson & Alibali, 1999) and thus are complementary. Students can have strong procedural knowledge but weaker conceptual understanding and vice versa (Hallett et al., 2010), but both are important.

The five arithmetic principles measured in the **Equations** tasks include the basic principles of *identity*, that is, adding or subtracting does not change the number; and *negation*, that is, subtracting a number from itself equals 0 (Robinson, 2017). The principle of inversion arises from an understanding of identity and negation, such that students can easily solve problems like 3 + 54 – 54 without calculating (Crooks & Alibali, 2014; Robinson & LeFevre, 2011). Two other principles, *commutativity* and *equivalence*, allow students to flexibly evaluate equations presented in a variety of formats. Commutativity is the principle that the order of operands in addition (and multiplication) doesn't matter, so that 3 + 4 will have the same answer as 4 + 3. Students implicitly understand this principle when they know they can solve both 2 + 5 and 5 + 2 by counting on from 5 (Siegler & Braithwaite, 2017).

Equivalence is the rule that the information on both sides of an equation must be equal. Equivalence is signalled by the equal sign and it is fundamental for learning algebra (Crooks & Alibali, 2014; Star & Rittle-Johnson, 2008). Knowledge of equivalence starts out very simply: for example, knowing that 4 = 4 is a valid equation. At first, students may think that the equal sign in a problem like 3 + 2 = __ means something like "add these two numbers and put the answer in the blank space." That operational notion must be replaced by a relational understanding—the equal sign really means "the quantity on the left side and the quantity on the right side should be the same". Students who acquire relational understanding earlier and therefore develop flexible problem solving skills will find fraction and algebra learning easier (Crooks & Alibali, 2014; McNeil et al., 2006; Prather & Alibali, 2009).

The **Equations** task will give teachers insight into which students are acquiring conceptual knowledge along with procedural skills. Conceptual knowledge is often implicit, in that students may not be able to describe their understanding, but it shows up when they can solve problems like 3 + 4 = __ + 3 without needing to do any arithmetic (Robinson, 2017; Robinson & LeFevre, 2011). Providing opportunities for students to see equations in many different forms and fostering a relational interpretation of the equal sign will support student progress.

Students in Grade 4 also complete the **Calculations** task which includes multi-digit addition and subtraction. This task measures students' computation skills and their understanding of place value. To solve these problems, students need to manipulate the numbers. They can use standard algorithms or other strategies such as breaking the number down (e.g., 83 + 27 = (7 + 3) and (80 + 20) = 110). As students' number skills develop, their solution strategies become more efficient (Hickendorff et al., 2019).

To quickly and efficiently solve calculation problems that involve regrouping, students need to process place-value information.

Children who experience math difficulties have trouble processing place value. Compared to their peers, these students take more time and make more errors when solving multi-digit arithmetic (Lambert & Moeller, 2019).

Students make different types of errors when completing multi-digit arithmetic. For example, consider the problem 32 – 18; students can make mis-calculation errors (32 – 18 = 15), subtract the smaller number from the larger (32 – 18 = 26), use the wrong operation (32 – 18 = 50), make regrouping errors or fail to regroup (32 – 18 = 24). By reviewing student errors, teachers can gauge the knowledge level of their students and target instruction accordingly. Students' performance on the calculation task will give teachers insight into their students' developing understanding of place-value, and their efficient use of calculation strategies.

The **Fractions** task measures student's understanding of the knowledge that underlies successful fraction skills and predict later mathematical achievement. The questions tap into three foundational fraction concepts: representation, equivalency, and magnitude.

When representing fractions, students may misunderstand what the numerator and denominator represent. For example, a student may incorrectly identify 3 out of 8 slices of pizza as $\frac{3}{5}$, $\frac{8}{3}$, or possibly $\frac{5}{8}$. When comparing fractions, students may misunderstand that the magnitude of the fraction is based on the relation between the numerator and denominator. Instead, they may rely on their understanding of whole numbers to compare fractions. For example, students may assume that a larger denominator indicates a larger fraction (e.g., thinking $\frac{1}{3}$, is greater than $\frac{1}{2}$), or a larger numerator indicates a larger fraction (e.g., thinking $\frac{5}{8}$ is greater than $\frac{2}{3}$), or they might look at the gap between the numerator and denominator and assume a smaller gap indicates a larger fraction (e.g., thinking $\frac{2}{3}$ is greater than $\frac{7}{9}$) (Rinne et al., 2017).

Student's foundational knowledge of fractions is predictive of their developing fraction skills, their later understanding of algebra, and their overall math achievement in high school (Booth & Newton, 2012; Jordan et al., 2017; Siegler et al., 2012). Helping students develop strong fraction skills, prepares them for later mathematical success.

To summarize, the Provincial Numeracy Screeners can be used to measure a broad range of number skills in children and students from Kindergarten to Grade 4. The number skills measured are based on cognitive research and rooted in theories of mathematical development. Tasks range from early verbal counting to more advanced arithmetic fluency. The results can be used by teachers generally, to judge the skill level within their classroom, and specifically, to identify students with weak number skills. Teachers can then customize and target their instruction to address gaps in their students' number knowledge.

References

Aunio, P., & Räsänen, P. (2016). Core numerical skills for learning mathematics in children aged five to eight years – a working model for educators. *European Early Childhood Education Research Journal, 24*(5), 684–704. https://doi.org/10.1080/1350293X.2014.996424

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science*, *17*(5), 775–785. https://doi.org/10.1111/desc.12155

Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, *42*(1), 189–201. https://doi.org/10.1037/0012-1649.41.6.189

Bugden, S., Peters, L., Nosworthy, N., Archibald, L., & Ansari, D. (2020). Identifying children with persistent developmental dyscalculia from a 2-min test of symbolic and nonsymbolic numerical magnitude processing. *Mind, Brain, and Education*, mbe.12268. https://doi.org/10.1111/mbe.12268

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General, 130*(2), 299–315. https://doi.org/10.1037/0096-3445.130.2.299

Carey, S., & Barner, D. (2019). Ontogenetic origins of human integer representations. *Trends in Cognitive Sciences,* in press, 1–13. https://doi.org/10.1016/j.tics.2019.07.004

Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using Measures of Number Sense to Screen for Difficulties in Mathematics: Preliminary Findings. *Assessment for Effective Intervention, 30*(2), 3–14. https://doi.org/10.1177/073724770503000202

Clayton, F. J., Copper, C., Steiner, A. F., Banfi, C., Finke, S., Landerl, K., & Göbel, S. M. (2020). Two-digit number writing and arithmetic in Year 1 children: Does number word inversion matter? *Cognitive Development*, *56*(April), 1–14. https://doi.org/10.1016/j.cogdev.2020.100967

Cowan, R., Donlan, C., Shepherd, D.-L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology*, *103*(4), 786–803. https://doi.org/10.1037/a0024556

Crooks, N. M., & Alibali, M. W. (2014). Defining and measuring conceptual knowledge in mathematics. *Developmental Review*, *34*(4), 344–377. https://doi.org/10.1016/j.dr.2014.10.001

DeStefano, D., & LeFevre, J.-A. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16*, 353–386. https://doi.org/10.1080/09541440244000328

Devlin, B. L., Jordan, N. C., & Klein, A. (2022). Predicting mathematics achievement from subdomains of early number competence: Differences by grade and achievement level. *Journal of Experimental Child Psychology, 217*, 105354. https://doi.org/10.1016/j.jecp.2021.105354

Di Lonardo Burr, S., Xu, C., Douglas, H., LeFevre, J.-A., & Susperreguy, M. I. (2022). Walking another pathway: The inclusion of relational reasoning in the pathways to mathematics model. *Journal of Educational Psychology*.

Friedman, N. P., & Miyake, A. (2017). Unity and diversity of executive functions: Individual differences as a window on cognitive structure. *Cortex 86*, 186–204. https://doi.org/10.1016/j.cortex.2016.04.023

Fyfe, E. R., Rittle-Johnson, B., & DeCaro, M. S. (2012). The effects of feedback during exploratory mathematics problem solving: Prior knowledge matters. *Journal of Educational Psychology*, *104*(4), 1094–1108. https://doi.org/10.1037/a0028389

Gallistel, C. R., & Gelman, R. (1990). The what and how of counting. In *Cognition* (Vol. 34, Issue 2, pp. 197–199). https://doi.org/10.1016/0010-0277(90)90043-J

Gasteiger, H., & Moeller, K. (2021). Fostering early numerical competencies by playing conventional board games. *Journal of Experimental Child Psychology 204*, 105060. https://doi.org/10.1016/j.jecp.2020.105060

Gray, S. A., & Reeve, R. A. (2014). Preschoolers' dot enumeration abilities are markers of their arithmetic competence. *PloS One*, *9*(4), e94428. https://doi.org/10.1371/journal.pone.0094428

Gray, S. A., & Reeve, R. A. (2016). Number-specific and general cognitive markers of preschoolers' math ability profiles. *Journal of Experimental Child Psychology*, *147*, 1–21. https://doi.org/10.1016/j.jecp.2016.02.004

Groen, G. J., & Parkman, J. M. (1972). A chronometric analysis of simple addition. *Psychological Review*, *79*(4), 329–343. https://doi.org/10.1037/h0032950

Hallett, D., Nunes, T., & Bryant, P. (2010). Individual differences in conceptual and procedural knowledge when learning fractions. *Journal of Educational Psychology*, *102*(2), 395–406. https://doi.org/10.1037/a0017486

Hawes, Z., Nosworthy, N., Archibald, L., & Ansari, D. (2019). Kindergarten children's symbolic number comparison skills predict 1st grade mathematics achievement: Evidence from a two-minute paper-and-pencil test. *Learning and Instruction, 59*(January 2018), 21–33. https://doi.org/10.1016/j.learninstruc.2018.09.004

Hickendorff, M., Torbeyns, J., & Verschaffel, L. (2019). Multi-digit addition, subtraction, multiplication, and division strategies. In *International Handbook of Mathematical Learning Difficulties: From the Laboratory to the Classroom* (pp. 543–560). https://doi.org/10.1007/978-3-319-97148-3_32

Hornburg, C. B., Schmitt, S. A., Purpura, D. J., & Hornburg, C. B. (2018). Relations between preschoolers' mathematical language understanding and specific numeracy skills. *Journal of Experimental Child Psychology*, *176*, 84–100. https://doi.org/10.1016/j.jecp.2018.07.005

Hurst, M. A., Anderson, U., & Cordes, S. (2017). Mapping among number words, numerals, and nonsymbolic quantities in preschoolers. *Journal of Cognition and Development*, *18*(1), 41–62. https://doi.org/10.1080/15248372.2016.1228653

Hutchison, J. E., Ansari, D., Zheng, S., De Jesus, S., & Lyons, I. M. (2022). *Extending ideas of numerical order beyond the count-list from kindergarten to first grade*. https://doi.org/10.1016/j.cognition.2022.105019

Imbo, I., Vanden Bulcke, C., De Brauwer, J., & Fias, W. (2014). Sixty-four or four-and-sixty? The influence of language and working memory on children's number transcoding. *Frontiers in Psychology, 5*(April), 1–10. https://doi.org/10.3389/fpsyg.2014.00313

Jiménez Lira, C., Carver, M., Douglas, H., & LeFevre, J.-A. A. (2017). The integration of symbolic and non-symbolic representations of exact quantity in preschool children. *Cognition*, 166, 382–397. https://doi.org/10.1016/j.cognition.2017.05.033

Kalra, P. B., Hubbard, E. M., & Matthews, P. G. (2020). Taking the relational structure of fractions seriously: Relational reasoning predicts fraction knowledge in elementary school children. *Contemporary Educational Psychology*, *62*, 101896. https://doi.org/10.1016/j.cedpsych.2020.101896

Kamawar, D., LeFevre, J.-A., Bisanz, J., Fast, L., Skwarchuk, S.-L., Smith-Chant, B., & Penner-Wilger, M. (2010). Knowledge of counting principles: How relevant is order irrelevance? *Journal of Experimental Child Psychology, 105*(1–2), 138–145. https://doi.org/10.1016/j.jecp.2009.08.004

Knuth, E. J., Stephens, A. C., Mcneil, N. M., & Alibali, M. W. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education, 37*(4), 297–312. http://www.jstor.org/stable/30034852

Lambert, K., & Moeller, K. (2019). Place-value computation in children with mathematics difficulties. *Journal of Experimental Child Psychology*, *178*, 214–225. https://doi.org/10.1016/j.jecp.2018.09.008

LeFevre, J.-A., DeStefano, D., Coleman, B., & Shanahan, T. (2005). Mathematical cognition and working memory. *Handbook of Mathematical Cognition*, 377–508. https://doi.org/10.1017/CBO9781107415324.004

LeFevre, J.-A., Fast, L., Skwarchuk, S. L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, 81(6), 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508

LeFevre, J.-A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S.-L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology*, *4*, 641. https://doi.org/10.3389/fpsyg.2013.00641

LeFevre, J.-A., & Liu, J. (1997). The role of experience in numerical skill: Multiplication performance in adults from Canada and China. *Mathematical Cognition*, *3*(1), 31–62. https://doi.org/10.1080/135467997387470

LeFevre, J.-A., Smith-Chant, B. L., Fast, L., Skwarchuk, S.-L., Sargla, E., Arnup, J. S., Penner-Wilger, M., Bisanz, J., & Kamawar, D. (2006). What counts as knowing? The development of conceptual and procedural knowledge of counting from kindergarten through Grade 2. *Journal of Experimental Child Psychology*, *93*(4), 285–303. https://doi.org/10.1016/j.jecp.2005.11.002

Lin, X. (2020). Investigating the unique predictors of word-problem solving using meta-analytic structural equation modeling. *Educational Psychology Review*. https://doi.org/10.1007/s10648-020-09554-w

Litkowski, E. C., Duncan, R. J., Logan, J. A. R., Purpura, D. J., & Litkowski, E. C. (2020). When do preschoolers learn specific mathematics skills? Mapping the development of early numeracy knowledge. *Journal of Experimental Child Psychology*, *195*, 104846. https://doi.org/10.1016/j.jecp.2020.104846

Lyons, I. M., & Ansari, D. (2015). Foundations of children's numerical and mathematical skills. In *Advances in child development and behavior* (1st ed., Vol. 48, pp. 93–116). Elsevier Inc. https://doi.org/10.1016/bs.acdb.2014.11.003

Lyons, I. M., & Beilock, S. L. (2013). Ordinality and the nature of symbolic numbers. *Journal of Neuroscience*, *33*(43), 17052–17061. https://doi.org/10.1523/JNEUROSCI.1775-13.2013

Lyons, I. M., Price, G. R., Vaessen, A., Blomert, L., & Ansari, D. (2014). Numerical predictors of arithmetic success in grades 1-6. *Developmental Science*, 17(5), 714–726. https://doi.org/10.1111/desc.12152

McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. E. (2006). Middle-school students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction*, *24*(3), 367–385. https://doi.org/10.1207/s1532690xci2403_3

Moura, R., Wood, G., Pinheiro-Chagas, P., Lonnemann, J., Krinzinger, H., Willmes, K., & Haase, V. G. (2013). Transcoding abilities in typical and atypical mathematics achievers: The role of working memory and procedural and lexical competencies. *Journal of Experimental Child Psychology*, *116*(3), 707–727. https://doi.org/10.1016/j.jecp.2013.07.008

Nosworthy, N., Bugden, S., Archibald, L., Evans, B., & Ansari, D. (2013). A Two-Minute Paper-and-Pencil Test of Symbolic and Nonsymbolic Numerical Magnitude Processing Explains Variability in Primary School Children's Arithmetic Competence. *PLoS ONE, 8*(7). https://doi.org/10.1371/journal.pone.0067918

Peng, P., Lin, X., Ünal, Z. E., Lee, K., Namkung, J., Chow, J., & Sales, A. (2020). Examining the mutual relations between language and mathematics: A meta-analysis. *Psychological Bulletin*, *146*(7), 595–634. https://doi.org/10.1037/bul0000231

Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: Moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology*, *108*(4), 455–473. https://doi.org/10.1037/edu0000079

Prather, R. W., & Alibali, M. W. (2009). The development of arithmetic principle knowledge: How do we know what learners know? *Developmental Review*, *29*(4), 221–248. https://doi.org/10.1016/j.dr.2009.09.001

Price, G. R., Mazzocco, M. M. M., & Ansari, D. (2013). Why mental arithmetic counts: Brain activation during single digit arithmetic predicts high school math scores. *Journal of Neuroscience*, *33*(1), 156–163. https://doi.org/10.1523/JNEUROSCI.2936-12.2013

Purpura, D. J., Baroody, A. J., & Lonigan, C. J. (2013). The transition from informal to formal mathematical knowledge: Mediation by numeral knowledge. *Journal of Educational Psychology*, *105*(2), 453–464. https://doi.org/10.1037/a0031753

Purpura, D. J., & Ganley, C. M. (2014). Working memory and language: Skill-specific or domain-general relations to mathematics? *Journal of Experimental Child Psychology*, *122*(1), 104–121. https://doi.org/10.1016/j.jecp.2013.12.009

Purpura, D. J., & Lonigan, C. J. (2015). Early Numeracy Assessment: The Development of the Preschool Early Numeracy Scales. *Early Education and Development*, *26*(2), 286–313. https://doi.org/10.1080/10409289.2015.991084

Purpura, D. J., & Reid, E. E. (2016). Mathematics and language: Individual and group differences in mathematical language skills in young children. *Early Childhood Research Quarterly*, *36*, 259–268. https://doi.org/10.1016/j.ecresq.2015.12.020

Purpura, D. J., Reid, E. E., Eiland, M. D., & Baroody, A. J. (2015). Using a Brief Preschool Early Numeracy Skills Screener to Identify Young Children With Mathematics Difficulties. *School Psychology Review*, *44*(1), 41–59. https://doi.org/10.17105/SPR44-1.41-59

Raghubar, K. P., Barnes, M. A., & Hecht, S. A. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*(2), 110–122. https://doi.org/10.1016/j.lindif.2009.10.005

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other? *Journal of Educational Psychology*, *91*(1), 175–189. https://doi.org/10.1037/0022-0663.91.1.175

Robinson, K. M. (2017). The Understanding of Additive and Multiplicative Arithmetic Concepts. In D. C. Geary, D. B. Berch, R. J. Ochsendorf, & K. Mann Koepke (Eds.), *Acquisition of Complex Arithmetic Skills and Higher-Order Mathematics Concepts* (pp. 21–46). Elsevier. https://doi.org/10.1016/B978-0-12-805086-6.00002-3

Robinson, K. M., & Dubé, A. K. (2012). Children's Use of Arithmetic Shortcuts: The Role of Attitudes in Strategy Choice. *Child Development Research*, *2012*, 1–10. https://doi.org/10.1155/2012/459385

Robinson, K. M., & LeFevre, J.-A. (2011). The inverse relation between multiplication and division: Concepts, procedures, and a cognitive framework. *Educational Studies in Mathematics*, *79*(3), 409–428. https://doi.org/10.1007/s10649-011-9330-5

Sasanguie, D., Lyons, I. M., De Smedt, B., & Reynvoet, B. (2017). Unpacking symbolic number comparison and its relation with arithmetic in adults. *Cognition*, *165*, 26–38. https://doi.org/10.1016/j.cognition.2017.04.007

Sasanguie, D., & Vos, H. (2018). About why there is a shift from cardinal to ordinal processing in the association with arithmetic between first and second grade. *Developmental Science*, 21(5), e12653. https://doi.org/10.1111/desc.12653

Schneider, M., Beeres, K., Coban, L., Merz, S., Susan Schmidt, S., Stricker, J., & De Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: a meta-analysis. *Developmental Science 20*(3), e12372. https://doi.org/10.1111/desc.12372

Siegler, R. S., & Braithwaite, D. W. (2017, January 3). Numerical development. In *Annual Review of Psychology*. Annual Reviews. https://doi.org/10.1146/annurev-psych-010416-044101

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., Susperreguy, M. I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*, *23*(7), 691–697. https://doi.org/10.1177/0956797612440101

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games-but not circular ones-improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, *101*(3), 545–560. https://doi.org/10.1037/a0014239

Simmons, F. R., Willis, C., & Adams, A.-M. (2012). *Different components of working memory have different relationships with different mathematical skills*. https://doi.org/10.1016/j.jecp.2011.08.011

Simsek, E., Jones, I., Hunter, J., & Xenidou-Dervou, I. (2021). Mathematical equivalence assessment: Measurement invariance across six countries. *Studies in Educational Evaluation*, *70*(November 2020), 101046. https://doi.org/10.1016/j.stueduc.2021.101046

Skwarchuk, S. L., & Anglin, J. M. (2002). Children's acquisition of the English cardinal number words: A special case of vocabulary development. *Journal of Educational Psychology*, *94*(1), 107–125. https://doi.org/10.1037/0022-0663.94.1.107

Sowinski, C., LeFevre, J.-A., Skwarchuk, S.-L., Kamawar, D., Bisanz, J., & Smith-Chant, B. L. (2015). Refining the quantitative pathway of the Pathways to Mathematics model. *Journal of Experimental Child Psychology*, *131*, 73–93. https://doi.org/10.1016/j.jecp.2014.11.004

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in problem solving: The case of equation solving. *Learning and Instruction*, *18*(6), 565–579. https://doi.org/10.1016/j.learninstruc.2007.09.01

Toll, S. W. M., & Van Luit, J. E. H. (2014). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children*, *81*(1), 64–78. https://doi.org/10.1177/0014402914532233

Vanbinst, K., Ansari, D., Ghesquière, P., & De Smedt, B. (2016). Symbolic Numerical Magnitude Processing Is as Important to Arithmetic as Phonological Awareness Is to Reading. *PLOS ONE*, *11*(3), e0151045. https://doi.org/10.1371/journal.pone.0151045

Xu, C., Lafay, A., Douglas, H., Di Lonardo Burr, S., LeFevre, J.-A., Osana, H. P., Skwarchuk, S., Wylie, J., Simms, V., & Maloney, E. A. (2021). The role of mathematical language skills in arithmetic fluency and word-problem solving for first- and second-language learners. *Journal of Educational Psychology*, 1–29. https://doi.org/10.1037/edu0000673

Xu, C., & LeFevre, J. A. (2016). Training young children on sequential relations among numbers and spatial decomposition: Differential transfer to number line and mental transformation tasks. *Developmental Psychology*, *52*(6), 854–866. https://doi.org/10.1037/dev0000124

Xu, C., & LeFevre, J.-A. (2021). Children's knowledge of symbolic number in grades 1 and 2: Integration of associations. *Child Development*, *92*(3), 1099–1117. https://doi.org/10.1111/cdev.13473

Xu, C., LeFevre, J.-A., Skwarchuk, S.-L., Di Lonardo Burr, S., Lafay, A., Wylie, J., Osana, H. P., Douglas, H., Maloney, E. A., & Simms, V. (2021). Individual differences in the development of children's arithmetic fluency from grades 2 to 3. *Developmental Psychology*, *57*(7), 1067–1079. https://doi.org/10.1037/dev0001220

Zhang, X., & Lin, D. (2015). Pathways to arithmetic: The role of visual-spatial and language skills in written arithmetic, arithmetic word problems, and nonsymbolic arithmetic. *Contemporary Educational Psychology*, *41*, 188–197. https://doi.org/10.1016/j.cedpsych.2015.01.005

Zippert, E. L., Douglas, A. A., & Rittle-Johnson, B. (2020). Finding patterns in objects and numbers: Repeating patterning in pre-K predicts kindergarten mathematics knowledge. *Journal of Experimental Child Psychology*, *200*, 104965. https://doi.org/10.1016/j.jecp.2020.104965